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An Analysis on Farm Size and Nonparametric Efficiency Measurement for Food Crops in Pakistan

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ABSTRACT

The objective of this study is to evaluate the effect of farm size on productivity and technical efficiency of three major food crops of Pakistan over the time period 1948-2011. In order to assess the technical efficiency of three major food crops i.e. wheat, rice and maize in Pakistan, most widely known Data Envelopment Analysis (DEA) technique is used. The result of the study reveals that 88% inefficiency is observed in case of wheat production. The efficiency result of Maize crop demonstrates a 9 % over-utilization of farm size. The mean technical efficiency of rice crop for variable and constant return to scale frontiers are 0.91 and 0.62, respectively. Hence, indicating the signs of productive inefficiency in rice crop too. Consequently, present study suggests that instead of increasing area under food crops, it is the need of the hour to adopt modernized agricultural technique. Moreover, farmers have to equip with new advancement in agriculture and have to create awareness of high yielding seed varieties of food crops.

Keywords: Data envelpopment analysis (DEA), Variable Return to scale technical effciciency (VRSTE). Constant return to scale technical efficiency (CRSTE).

INTRODUCTION

“Hunger is exclusion – exclusion from the land, from income, jobs, wages, life and citizenship. When a person gets to the point of not having anything to eat, it is because all the rest has been denied. This is a modern form of exile. It is death in life…” Josue de Castro.

Agriculture is the second largest sector accounting for over 21 percent of GDP, and has remained by far the largest employer, absorbing 45 percent of the country’s labor force and 60 percent of the rural population that is directly or indirectly linked with agriculture for their livelihood.1 Its role is imperative ensuring food security, generating overall economic growth, reducing poverty and the transforming towards industrialization in Pakistan. Agriculture sector has strong forward and backward linkages with the rest of the economy, though not fully captured in the statistics. On the one hand, this sector is a primary supplier of raw materials to downstream industry and contributing substantially to Pakistan’s exports, on the other. It provides a large market for industrial products such as fertilizer, pesticides, tractors and agricultural implements and has remained a base for industrial development in Pakistan.

In order to stabilize the food crop prices and enhance the productivity of domestic food crop along with efficiency of producers the government of Pakistan has profoundly supported the food farming sector. Specifically, this support is given in the form of subsidized farm inputs for agricultural production. Besides, the consumer prices of food are being kept lower than the world market prices to facilitate the cheap provision of food crop to customers. In that context, the fluctuations in food prices drastically hit the Pakistan’s economy by worsening country’s real income position on international front. According to Aslam et. al., (2012), “the price volatility generates extra risks and is a particular

[1] According to Economic Survey of Pakistan (2011-2012).

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burden for low income producers who are least able to circumvent against these fluctuations, as well as for poor consumers. Increased volatility tends to lead to greater government intervention in agricultural markets often with sizeable fiscal costs". Due to the costs of subsidization and the significance of food crops as a food supply and the threat of food shortage, self-sufficiency in food crops has remained one of the essential national objectives.

According to Ahmad (2002), "farm sector in less developed countries like Pakistan are generally considered to play a vital role in the eradication of poverty. Pakistan was listed among the 40 countries facing food price crises at the time". The price of almost every agricultural commodity rose notably over the past few years. The prices of main food staples like wheat, maize and rice are rising rapidly. Both demand and supply side factors play a detrimental role in affecting food prices. On the supply side, subsidies and world food prices are considered to be the most important factors, whereas on demand side money supply, net importer/exporter position of coutry has remained the main source of the rising food prices.

As stated in Economic Survey (2011-2012), "flooding in 2011, affected crops like rice, cotton and sugarcane, although in the current year, 2011-12, they performed well and provided support and continued to support food security objectives this year. The agriculture sector recorded a growth of 3.1 percent in 2011-12. The profitability of agriculture sector during 2011-12, remained high because the farmers received good prices for rice, cotton and sugarcane, which allowed for greater financial resources passed on to the rural economy". Inspite of this, being an agricultural country Pakistan has not been strongly able to attain self sufficiency in food crop throughout the history even with increasing land area set into food production on annual basis. Various studies including Javed et. al., (2009), Hassan (2004), Ahmad (2002) and Ahmad & Ahmad (1998) have examined the low productivity and technical efficiency of food crops, and concluded that the technical inefficiency is one of major constraint to the rapid growth of food production and actual yield of major crop is much lower as compared with their potential yields, consequently.

The endeavor of this paper is to evaluate the technical efficiency of food crop in Pakistan from 1948-2011 for selected items namely, wheat, maize and rice. Farm size and productivity, having one of the most multifarious and prudence relation, is expected to be supportive regarding policies for agricultural development in Pakistan.

The rest of the paper is organized as follows. Next section deals with the data and methodology. And the 3rd section discusses results in detail, followed by conclusion and some policy recommendations.

DATA AND METHODOLOGY

Background and History of DEA

The Data Envelopment Analysis (DEA) was first developed by Charnes, Cooper and Rhodes under the assumption of constant returns to scale in 1978. In their originating study, they described DEA as a "mathematical programming model applied to observational data that provides a new way of obtaining empirical estimates of relations, such as the production functions and/or efficient production possibility surfaces that are cornerstones of modern economics".

Formally, DEA is a methodology directed to frontiers rather than central tendencies. Instead of trying to fit a regression plane through the center of the data as in statistical regression, non-parametric approaches do not require such restrictions, although they assume the absence of measurement or sampling errors and deviations from the production frontier are under the control of the production unit being considered. "Non-parametric methods, as originally conceived by Farell, 1957 used the unit input output space to create a frontier isoquant within the production possibility set. The frontier was determined by a single or a convex combination of efficient units which were then compared against inefficient units to calculate the extent of inefficiency. This method was later applied to the multiple input output case" (Murillo and Zamorano, 2004). DEA has been used to judge performance of non-profit organizations, hospitals, courts, school, colleges, universities, public sector and agriculture (Coelli, 1996). However, now a day's researcher also applied it to examine the performance of profit organizations.

Application of DEA on Pakistan's Major Crops

In this paper we apply the Data Envelopment Analysis (DEA) instrument in order to assess the technical efficiency of three major food crops of Pakistan. The rationale we merely use DEA and not Stochastic Frontier Analysis (SFA), is that these parametric methods typically involve the specification of functional forms. This function requires complexities in estimation. In our present case, we don't have farm-level data on multi inputs from 1948 to 2011 of major food crops of Pakistan, as the economic survey and MINFA only contains land area statement input variable and production output data. Supplementary input/output data are indispensable if one year to estimate a parametric functional form (Christensen and Greene, 1976) and (Seiford and Thrall, 1990). Herewith, this study will enable us to identify the years when Pakistan's major crops operated on their efficient frontier. If the yearly input-output combination lies on the DEA frontier, that mean on that year crop production is operating on production frontier.

Estimation Technique

In our yearly DEA estimations, we use a single-output/single-input for major crops technical efficiency measurement. We consider one input land area (measured by the total land area in Pakistan in hectors). The output vector we retains major crops production. The present study uses a single-step methodology. In the first step, data envelopment analysis (DEA) is used to model efficiencies as an explicit function of discretionary variables. The methodology is specified below.

"Assuming we have data on K inputs and M outputs of N years, xi is an input vector for the ith farm and yi is an output vector for the ith year. The K x N input matrix, X, and M x N output matrix, Y, represent the data of all for N crops. For each farm, we obtain a measure of the ratio of all outputs over all inputs, such as u/yi/v/xi, where u is an M * 1 vector of output weights and v is K * 1 vector of input weights. To select optimal weights we solve the mathematical programming problem as specified by Coelli, et al (1998)". Firstly,

$$\max_{u,v} (u/yi/v/xi) \tag{1.1}$$

subject to

$u/yj/v/xj \leq 1$, j= 1,2,……N,

$u, v \geq 0$

$$\max_{u,v} (u/yi/v/xi) \tag{1.2}$$

Subject to

$V/xi = 1$

$u/yj/v/xj \leq 1$, j= 1,2,……N,

$u, v \geq 0$

Secondly,

$$\min_{\theta, \lambda} \theta, \tag{1.3}$$

Subject to

$-yi + Y\lambda \geq 0$

$\theta xi - X\lambda \geq 0$

$\lambda \geq 0$

Where, θ = is a scalar, Restriction: $\theta \leq 1$, λ = is a Nx1 vector of constants

Coelli, et al (1998) suggest that a constant returns to scale DEA model is only appropriate when all firms are operating at an optimal scale, this is not possible in agriculture due to many constraints such as imperfect competition and financial constraints, etc. Bankers, et al. (1984) modifies the constant returns to scale DEA model into a variable returns to scale model by adding convexity constraints. Variable Returns to Scale (VRS) DEA model is also found in detail in the studies; Ferrier, D. and Lovell C, A. K., (1990), and Sharma et. al., 1999. Following Coelli, et al (1998), an input-oriented variable returns to scale DEA model will be used to estimate technical efficiency. Here, the objective is to determine the relative efficiency for each year. Efficiency is measured by the ratio of inputs to outputs as follows:

Min θ, λ θ, (1.4)

Subject to

$-y_i + Y\lambda \geq 0$

$x_i - X\lambda \geq 0$

$N1/\lambda = 1$

$\lambda \geq 0$

Where, N1/ λ = 1 represents a convexity constraint which ensures that an inefficient crop production year is only benchmarked against farms of a similar size. Y represents the output matrix for N year crop production. θ represents the total technical efficiency of the ith year. λ represents N x 1 constants. X represents the input matrix for N year crop production.

While in variable return to scale Data Envelopment Analysis we use the term pure technical efficiency because it is free from scale effects. DEA more flexible in case of variable return to scale and CRS and VRS carried out on the same data set. The ratio between CRS and VRS technical efficiency scores is called scale efficiency. A decision making unit is called scale efficient if VRS and CRS, technical efficiency score are equal. This relationship is identified as follows,

Scale efficiency (SE) = CRSTE/VRSTE.

Moreover, as the scale inefficiency can be due to the existence of either increasing or decreasing returns to scale. The relationship can be used to measure scale efficiency (SE) of ith year as: where SE = 1(CRSTE = VRSTE) implies scale efficiency and if CRS and SE < 1 this indicates scale inefficiency. Now, we turn to results and discussion based on the above given methodology.

RESULTS AND DISCUSSION

This section yields the findings for Constant Return to Scale Technical Efficiency (CRSTE), Variable Return to Scale Technical Efficiency (VRSTE) and Scale Efficiency (SE), next to the panorama of Return to Scale (RTS) for three major crops of Pakistan over the period 1948-2011.

Wheat Crop Efficiency

Wheat is the basic staple food for most of the population and largest grain source of the country. According to Economic Survey (2012), wheat contributes 12.5 percent to the value added in agriculture and 2.6 percent to GDP. However, the yield per hectare in 2011-12 posted a negative growth of 4.2 percent as compare to 11 percent growth last year due to standing water and other climatic factors. Overall, the results show that only one year i.e., 1953 out of 64 years the wheat production is found to be overall technically efficient with score equal to 1. This efficient year in wheat crop production define the efficient frontier of wheat production from year 1948-2011 and thus from the reference set for inefficient wheat crop production years. The remaining 63 years of wheat crop production can be determined as the radial distance from the production frontier.

Table1. *Technical Efficiency Frequency Distribution of Wheat Crop*

Wheat Frequency Distribution						
Freq	CRSTE	CRSTE%	VRSTE	VRSTE%	S.E	SE%
0.01-0.20	0	0	0	0	0	0
0.21-0.40	31	48.4	0	0	24	37.5
0.41-0.50	7	10.9	0	0	9	14.1
0.51-0.60	6	9.4	0	0	6	9.4
0.61-0.70	3	4.7	0	0	6	9.4
0.71-0.80	10	15.6	12	18.8	2	3.1
0.81-0.90	6	9.4	26	40.6	10	15.6
0.91-1.00	1	1.6	26	40.6	7	10.9
Mean	0.50	100	0.88	100	0.57	100
SD	0.22		0.08		0.24	
Max	1		1		1	
Min	0.23		0.72		0.25	
Total	64	100		100	64	100

It is prudent to mention that the process of land utilization in the wheat production frontier is not functioning quite well and the waste of land resources can lead to further inefficiency in wheat production. Consequently in upcoming years Pakistan could not move towards efficient production frontier until it start imitating the best land utilization approach and favorable input-output combination under wheat crop.

Table2. *Return to Scale in Wheat Production*

Return to Scale	N	Percentage
IRS	11	17.2
DRS	51	79.7
CRS	2	3.1
Total	64	100.0

Table 2 presents the frequency distribution of CRSTE, VRSTE and SE scores with descriptive analysis. The figures demonstrate that CRSTE score in wheat production ranges in between 0.2 to 1. The minimum technical efficiency is scoring 0.23 in year 2007 and 2009, while maximum 1 in year 1953. The average technical efficiency in CRSTE is 0.50 and standard deviation (SD) 0.22. On the other hand, the VRSTE scores ranges between 0.7 to 1. The maximum scoring 1 was in a number of years; 1953, 1966, 1967, 1969, 1987, 1993, and 2010. The mean VRSTE scores is 0.88 and standard deviation (SD) 0.08. So, overall technical inefficiency from 1948 to 2011 is found 12 percent in case of wheat production in Pakistan. Thus we may possibly conclude that the same level wheat output could be produced with 12 percent lesser use of area under wheat production.

Table 2 also yields the mean scale efficiency of wheat production from 1948 to 2011. If we perceived as a whole the mean value of scale efficiency is 0.57 with standard deviation of 0.24. Scale efficiency scores ranges from a minimum of 0.25 to maximum of 1. This result demonstrates that scale inefficiency is about 0.43 percent in wheat production. Only in the year 1953 wheat crop attained scale efficiency score equal to 1. So this can perceive from the results that the wheat crop in Pakistan did not operate at most productive scale size in the given period.

Table 3 shows that the wheat crop has been operating with some degree of scale inefficiency. Almost 51 years wheat crop face the decreasing return to scale (DRS) and 11 year countenance increasing return to scale (IRS) and only one year constant return to scale (CRS).The results in wheat crop suggest that the degree of scale efficiency was found to be lower than the degree of VRS technical efficiency which designated that the fraction of overall inefficiency is due to producing at an inefficient scale relatively producing lower the production frontier. The results reveal that there is immense need to produce more wheat by escalating productivity and efficiency.

Table3. *Technical Efficiency Frequency Distribution of Maize Crop*

Maize Frequency Distribution						
Freq	CRSTE	CRSTE%	VRSTE	VRSTE%	S.E	SE%
0.01-0.20	0	0	0	0	0	0
0.21-0.40	7	10.9	0	0	4	6.3
0.41-0.50	2	3.1	2	3.1	1	1.6
0.51-0.60	8	12.5	0	0	11	17.2
0.61-0.70	13	20.3	0	0	9	14.1
0.71-0.80	11	17.2	3	4.6	7	10.9
0.81-0.90	18	28.1	20	31.1	12	18.8
0.91-1.00	5	7.8	39	60.9	20	31.3
Mean	0.69	100	0.91	100	0.76	100
SD	0.20		0.11		0.20	
Max	1		1		1	
Min	0.25		0.43		0.26	
Total	64	100	64	100	64	100

Maize Crop Efficiency

Table 4 to 6 reports the results on efficiency measurement of maize crop. The mean CRSTE in maize crop across all years is 0.69 with the 0.20 percent standard deviation as depicted from Table 5. The minimum CRSTE is about 0.25 in year 2011 and maximum level is 1 in year 1953. The mean VRSTE from 1948-2011 is 0.91 with minimum level of 0.43 in year 2011 and maximum level of 1 in several years. It is also divulge that about 39 years the VRSTE ranged from 0.91 to 1 and 20 years scoring in between 81 to 90. The results show that in Pakistan on average 9 percent over-use of farm size is observed from 1948-2012. In order to operate on the production frontier there is need to decrease 9 percent farm size under maize crop. No doubt scale economies at farm level play an imperative role in take over the activity. Similarly, Table 6 shows that out of 64 years maize production face DRS in 59 years. It depicts that Pakistan in maize production possessing a DRS at large structure.

Table4. *Return to Scale in Maize Production*

Return to Scale	N	Percentage
IRS	3	4.7
DRS	59	92.2
CRS	2	3.1
Total	64	100.0

Table5. *Technical Efficiency Frequency Distribution of Rice Crop*

Rice Frequency Distribution						
Freq	CRSTE	CRSTE%	VRSTE	VRSTE%	S.E	SE%
0.01-0.20	0	0	0	0	0	
0.21-0.40	7	10.9	0	0	4	6.3
0.41-0.50	16	25.0	0	0	10	15.6
0.51-0.60	19	29.7	0	0	20	31.3
0.61-0.70	2	3.1	3	4.7	4	6.3
0.71-0.80	1	1.6	2	3.1	4	6.3
0.81-0.90	7	10.9	24	37.5	5	7.8
0.91-1.00	12	18.8	35	54.7	17	26.6
Mean	0.62	100	0.66	100	0.35	100
SD	0.22		0.08		0.22	
Max	1		1		1	
Min	0.35		0.66		0.35	
Total	64	100	64	100	64	100

Table6. *Return to Scale in Rice Production*

Return to Scale	N	Percentage
IRS	4	6.3
DRS	49	76.6
CRS	11	17.2
Total	64	100.0

Rice Crop Efficiency

Rice ranks as second amongst staple food grain crops in Pakistan and it has been a major source of foreign exchange earnings in recent years. Rice accounts 4.9 percent of the value added in agriculture and 1.0 percent of GDP. The production has increased as compare to last year due to increase in the area sown. The yield per hectare has improved in year 2011-12 as compare to last year (Pakistan, Govt. of, 2012).

In case of rice crop production from 1948-2011, Table 7 shows that the mean technical efficiency for variable return to scale and constant return to scale frontier are 0.91 and 0.62, respectively, so results of DEA analysis shows the signs of productive inefficiency in rice crop too. In requisites of the constant return to scale model, out of 64 years only one year is fully efficient while under variable return to scale model, 6 out of the 64 years are fully efficient.

Correspondingly Table 8 demonstrates the descriptive analysis and frequency distribution. Results simply show that CRSTE are equal or less than those resulting from the VRSTE model. The mean scale efficiency for the rice production from 1964-2011 is 0.69. Out of the 64 years, 11 years show constant return to scale, 49 years show increasing returns to scale, while only 4 years shows decreasing return to scale. Thus table 9 substantiates that Pakistan in rice production owing a decreasing return to scale at large structure. The inefficiency in rice production in Pakistan from 1964-2011 proves that there is significant room for an increase in rice production without extra farm size.

CONCLUSIONS AND POLICY IMPLICATIONS

At the time of independence, total population of Pakistan was 32.5 million which has reached 176.6 million in 2011. As compare to rapidly growing population the pace of food crop productivity has not been up to the mark to fulfill the dietary needs of the growing segment of population, it is necessary to improve efficiency and productivity of food crops to plug the gap between supply and demand. Instead of investing scarce resources of Pakistan to import wheat and other food items as recommended by Muhammad (2010).

According to Javed et al, (2009), measuring technical efficiency in developed and developing country is still an area of research. It is very important especially in case of developing countries like Pakistan where potential to increase the production through extension in cultivated area and adoption of new technology is limited.

The result of the study indicates that in case of all food crops Pakistan is not technically efficient throughout the history. Food production although increased in Pakistan from 1948-2011, but overall yield could not meet the optimum potential level due to technical inefficiencies. Overall, wheat crop technical inefficiency from 1948 to 2011 is found 88 percent in case of wheat production in Pakistan. Thus we may possibly conclude that the same level wheat output could be produced with 12 percent lesser use of area under wheat production.

The result of maize crop indicates that out of 64 years maize production face decreasing return to scale (DRS) in 59 years. It depicts that Pakistan in maize production possesses a DRS at large structure. The inefficiency in rice production in Pakistan from 1948-2011 pointed towards the significant margin for an increase in rice production without extra involvement of farm size. The result indicates that Pakistan has remained technically inefficient in all major food crops and is suffering from reduced profit from 1948-2011. Such inefficiency leads ultimately to less competitiveness of Pakistan in case of food crops.

Therefore, there is need to improve the technical efficiency in order to shift production function upward. Instead of increasing area under food crops, it is need of the time to adopt land-saving technologies that is supported with modernized agricultural technique, farmer have to cope with new advancement in agriculture and should create awareness of high yielding varieties of food crops. In new growing season government should have considerations about regulations to curb fake supply of food crop seeds selling in the country and it is also the need of time to solicit for the issuance of an ordinance to ensure the supply of quality of recommended seeds/varieties to the farmers.

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